積分計算

問題 4-1

1

(1) $\int\left(x^8+\frac{1}{x^3}\right)dx=\int x^8dx+\int x^{-3}dx=\frac{1}{1+8}x^{1+8}+\frac{1}{1-3}x^{1-3}+C=\frac{x^9}{9}-\frac{1}{2x^2}+C$

(2) $\int\frac{(x+1)^3}{x^3}dx=\int\left(1+\frac{1}{x}\right)^3dx=\int\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)dx=x+3\ln|x|-\frac{3}{x}-\frac{1}{2x^2}+C$

(3) $\int(1+x)\sqrt{x}dx=\int\left(x^{1/2}+x^{3/2}\right)dx=\frac{1}{1+1/2}x^{1+1/2}+\frac{1}{1+3/2}x^{1+3/2}+C=\frac{2}{3}x^{3/2}+\frac{2}{5}x^{5/2}+C$

(4) $\int(2\sin x+5\cos x)dx=2\int\sin xdx+5\int\cos xdx=-2\cos x+5\sin x+C$

問題 4-2, 1.

(1) $t=x+1,\ dt=dx,\ \int(x+1)^5dx=\int t^5dt=\frac{1}{6}t^6+C=\frac{1}{6}(x+1)^6+C$

(2) $t=x^3+2,\ dt=3x^2dx,\ \int x^2(x^3+2)^4dx=\int(x^3+2)^4\frac{3x^2dx}{3}$
$=\int t^4\frac{dt}{3}=\frac{1}{3\cdot5}t^5+C=\frac{1}{15}(x^3+2)^5+C$

(3) $t=6x+8, dt=6dx, \int\cos(6x+8)dx=\frac{1}{6}\int\cos tdt=\frac{1}{6}\sin(6x+8)+C$

(4) $t=\sin x,\ dt=\cos xdx,\ \int\sin^2x\cos xdx=\int t^2dt=\frac{1}{3}t^3+C=\frac{1}{3}\sin^3x+C$

(5) $t=1-2x^2,\ dt=-4xdx,\ \int x\sqrt{1-2x^2}dx=\int\sqrt{t}\frac{dt}{-4}=\frac{t^{3/2}}{-4\cdot\frac{3}{2}}+C=-\frac{1}{6}\left(1-2x^2\right)^{3/2}+C$

(6) $t=\frac{1}{a}x,\ \int\frac{dx}{x^2+a^2}=\int\frac{a\,dt}{a^2\left(t^2+1\right)}=\frac{1}{a}\int\frac{dt}{t^2+1}=\frac{1}{a}\arctan t+C=\frac{1}{a}\arctan\left(\frac{x}{a}\right)+C$

ただし積分公式，80 ページ（9）を使用

問題 4-2, 2.

(1)

$$u=x, v'=\sqrt{1+x} \Rightarrow u'=1, v=\int \sqrt{1+x}dx=\frac{2}{3}(1+x)^{3/2}$$

$$\int x\sqrt{1+x}dx=\frac{2}{3}x(1+x)^{3/2}-\int 1\cdot\frac{2}{3}(1+x)^{3/2}dx=\frac{2}{3}x(1+x)^{3/2}-\frac{2}{3}\frac{2}{5}(1+x)^{5/2}+C$$

$$=\frac{2}{3}(1+x)^{3/2}\left(x-\frac{2}{5}(1+x)\right)+C=\frac{2}{3}(1+x)^{3/2}\left(\frac{3}{5}x-\frac{2}{5}\right)+C=\frac{2}{15}(1+x)^{3/2}(3x-2)+C$$

(2)

$$u=x, v'=\cos x \Rightarrow u'=1, v=\sin x,$$

$$\int x\cos xdx=x\sin x-\int 1\cdot\sin xdx=x\sin x+\cos x+C$$

(3)

$$u=\ln x, v'=x^2 \Rightarrow u'=\frac{1}{x},\ v=\frac{1}{3}x^3,$$

$$\int x^2\ln xdx=\frac{1}{3}x^3\ln x-\int\frac{1}{3}x^2dx=\frac{1}{3}x^3\ln x-\frac{1}{9}x^3+C=\frac{1}{9}x^3(3\ln x-1)+C$$

(4)

$$u=\ln(x^2+4), v'=1 \Rightarrow u'=\frac{2x}{x^2+4}, v=x$$

$$\int\ln(x^2+4)dx=x\ln(x^2+4)-\int\frac{2x}{x^2+4}xdx$$

$$\int\frac{2x}{x^2+4}xdx=2\int\frac{x^2+4-4}{x^2+4}dx=2\int dx-2\int\frac{4}{x^2+4}dx=2x-8\frac{1}{2}\arctan\frac{x}{2}$$

$$\int\ln(x^2+4)dx=x\ln(x^2+4)-2x+4\arctan\frac{x}{2}+C$$

(5)

$$u=x^2, v'=e^x \Rightarrow u'=2x, v=e^x$$

$$\int x^2e^xdx=x^2e^x-\int 2xe^xdx$$

$$u=x, v'=e^x \Rightarrow u'=1, v=e^x$$

$$\int xe^xdx=xe^x-e^x+\tilde{C}$$

$$\int x^2e^xdx=x^2e^x-2(xe^x-e^x+\tilde{C})=x^2e^x-2xe^x+2e^x+C=e^x\left(x^2-2x+2\right)+C$$

(6)教科書どおりで言うことなし .....

x = tan θ

必要なら対応箇所の「板書メモ No.2」を参照すること