4章演習問題 [4] 具体例

$$t=\sqrt{x^2-x+2}+x$$

$$t-x=\sqrt{x^2-x+2}$$

$$t^2-2tx+x^2=x^2-x+2$$

$$t^2=x(2t-1)+2$$

$$x=\frac{t^2-2}{2t-1}$$

$$dx=\frac{(2t)(2t-1)-2(t^2-2)}{(2t-1)^2}dt$$

$$=\frac{2(t^2-t+2)}{(2t-1)^2}dt$$

$$I=\int\frac{dx}{x\sqrt{x^2-x+2}}$$

$$\begin{cases} f(x,y)=\dfrac{1}{xy}, \\ y=\sqrt{x^2-x+2} \end{cases}$$

$$I=\int\frac{\left(\dfrac{2(t^2-t+2)}{(2t-1)^2}\right)dt}{\left(\dfrac{t^2-2}{2t-1}\right)\left(t-\left(\dfrac{t^2-2}{2t-1}\right)\right)}=\int\frac{\left(\dfrac{2(t^2-t+2)}{(2t-1)^2}\right)dt}{\left(\dfrac{t^2-2}{2t-1}\right)\left(\dfrac{2t^2-t}{2t-1}-\left(\dfrac{t^2-2}{2t-1}\right)\right)}$$

$$=\int\frac{\left(\dfrac{2(t^2-t+2)}{(2t-1)^2}\right)dt}{\left(\dfrac{t^2-2}{2t-1}\right)\left(\dfrac{t^2-t+2}{2t-1}\right)}=\int\frac{2}{t^2-2}dt$$

問題4-2 (演習[4]と類似) 具体例

$$I=\int\frac{1-2\cos x}{5-4\cos x}dx$$

$$f(\sin x,\cos x)=\frac{1-2\cos x}{5-4\cos x}$$

$$t=\tan\frac{x}{2},\quad dt=\frac{dx}{2\cos^2\frac{x}{2}}=\frac{1}{2}\left(1+\tan^2\frac{x}{2}\right)dx=\frac{1}{2}\left(1+t^2\right)dx,$$

$$dx=\frac{2dt}{1+t^2}$$

$$\cos x=\cos\left(2\cdot\frac{x}{2}\right)=2\cos^2\frac{x}{2}-1=\frac{2}{\tan^2\frac{x}{2}+1}-1=\frac{1-\tan^2\frac{x}{2}}{\tan^2\frac{x}{2}+1}=\frac{1-t^2}{1+t^2}$$

$$\frac{1-2\cos x}{5-4\cos x}dx=\frac{1-2\frac{1-t^2}{1+t^2}}{5-4\frac{1-t^2}{1+t^2}}\frac{2}{1+t^2}dt=\frac{\left(1+t^2\right)-2(1-t^2)}{5(1+t^2)-4(1-t^2)}\frac{2}{1+t^2}dt$$

$$=2\frac{3t^2-1}{\left(9t^2+1\right)\left(1+t^2\right)}dt=\frac{dt}{1+t^2}-\frac{3dt}{1+9t^2}$$

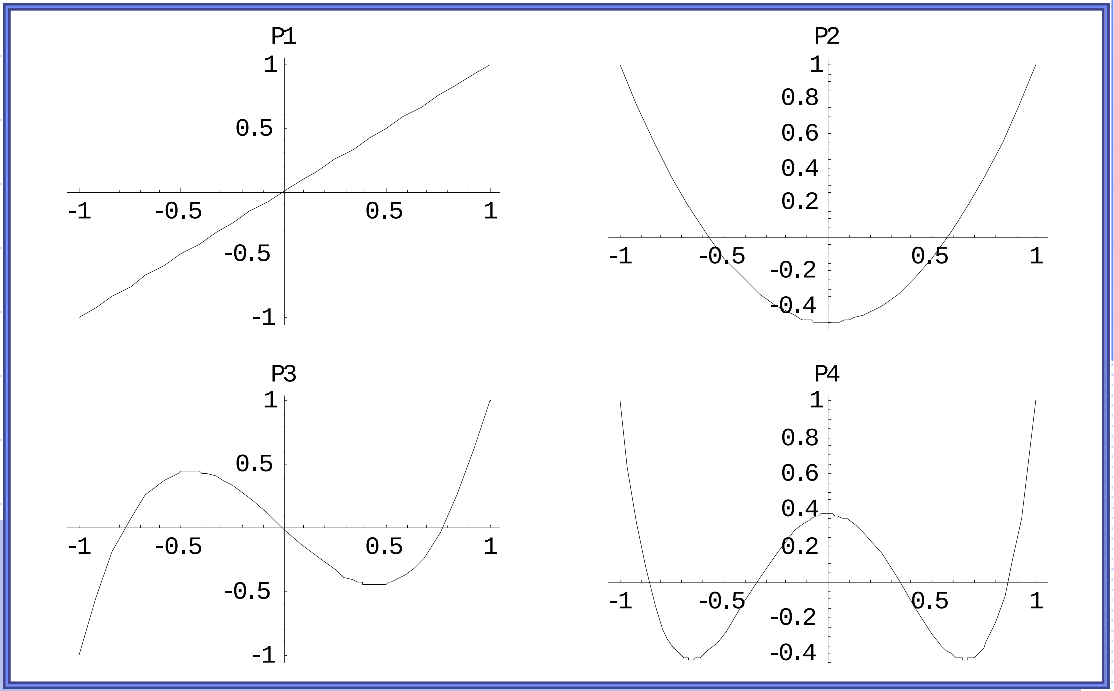
$$I=\arctan t-\arctan 3t+C$$

$$=\arctan\left(\tan\frac{x}{2}\right)-\arctan\left(3\tan\frac{x}{2}\right)+C$$

$$=\frac{x}{2}-\arctan\left(3\tan\frac{x}{2}\right)+C$$

4章演習問題[9] No1

P_nの具体的な形



$$P_0 = \frac{1}{2^0 0!} 1 = 1$$

$$P_1 = \frac{1}{2^1 1!} \frac{d}{dx}\left(x^2 - 1\right) = \frac{1}{2} 2x = x$$

$$P_2 = \frac{1}{2^2 2!} \frac{d^2}{dx^2}\left(x^2 - 1\right)^2 = \frac{1}{8} 2 \frac{d}{dx}\left[\left(x^2 - 1\right) 2x\right] = \frac{1}{2}(3x^2 - 1)$$

$$P_3 = \frac{1}{2}\left(5x^3 - 3x\right), P_4 = \frac{1}{8}\left(35x^4 - 30x^2 + 3\right), P_5 = \frac{1}{8}\left(63x^5 - 70x^3 + 15x\right)$$

4章演習問題[9]
No2

直交関係の具体例

$$\int_{-1}^{1} P_1 \cdot 1dx = \int_{-1}^{1} xdx = 0,$$

$$\int_{-1}^{1} P_2 \cdot 1dx = \int_{-1}^{1} \left(3x^2 - 1\right)dx = \left[x^3 - x\right]_{-1}^{1} = 0,$$

$$\int_{-1}^{1} P_2 \cdot xdx = \int_{-1}^{1} \left(3x^3 - x\right)dx = 0$$

前ページのグラフを見よ

ルジャンドル多項式は
偶(奇)数次は偶(奇)関数

理由:
偶関数の導関数は奇関数
奇関数の導関数は偶関数

偶関数と奇関数の積は奇関数

奇関数の積分[-1, 1]はゼロ

4章演習問題[9]

No3

ライプニッツの公式(3章演習問題[3])を用いる

$$\frac{d^m}{dx^m}\left(x^2-1\right)^n=\left[(x+1)^n(x-1)^n\right]^{(m)}$$

$$=\sum_{k=0}^{m}\left({}_mC_k\right)\left[(x+1)^n\right]^{(m-k)}\left[(x-1)^n\right]^{(k)}$$

$$=\sum_{k=0}^{m}\left({}_mC_k\right)\left[n(n-1)\cdots(n-(m-k)+1)(x+1)^{n-(m-k)}\right]$$

$$\times\left[n(n-1)\cdots(n-k+1)(x-1)^{n-k}\right]$$

$$=\sum_{k=0}^{m}\left({}_mC_k\right)\frac{n!}{\left(n-(m-k)\right)!}\frac{n!}{(n-k)!}(x+1)^{n-(m-k)}(x-1)^{n-k}$$

4章演習問題[9]

No 4 ルジャンドル関数を書き下す

$$P_n = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$$

$$= \frac{1}{2^n n!}\sum_{k=0}^{n}\left({}_nC_k\right)\frac{n!}{\left(n-(n-k)\right)!}\frac{n!}{(n-k)!} \times (x+1)^{n-(n-k)}(x-1)^{n-k}$$

$$= \frac{1}{2^n n!}\sum_{k=0}^{n}\left({}_nC_k\right)\frac{n!}{k!}\frac{n!}{(n-k)!}(x+1)^k(x-1)^{n-k}$$

$$= \frac{1}{2^n}\sum_{k=0}^{n}\left({}_nC_k\right)^2(x+1)^k(x-1)^{n-k}$$

次数がnのとき，
n次の多項式

$$P_n(1) = \frac{1}{2^n}\left(\frac{\left({}_nC_0\right)^2(1+1)^0(1-1)^n+}{0} \cdots + 0 + \left({}_nC_n\right)^2(1-1)^0(1+1)^n\right)$$

$$=1$$

$$P_n(-1) = \frac{1}{2^n}\left(\left({}_nC_0\right)^2(-1-1)^n + \cdots + 0\right)$$

$$=\left(-1\right)^n$$

4章演習問題[9]
No5　(1) の部分積分　その1

$$\int_{-1}^{1}\left(\frac{d^n}{dx^n}(x^2-1)^n\right)x^k dx=\int_{-1}^{1}\left(\frac{d^{n-1}}{dx^{n-1}}(x^2-1)^n\right)' x^k dx$$

$$=\left[x^k\frac{d^{n-1}}{dx^{n-1}}(x^2-1)^n\right]_{-1}^{1}-\int_{-1}^{1}kx^{k-1}\frac{d^{n-1}}{dx^{n-1}}(x^2-1)^n dx=-k\int_{-1}^{1}\left(\frac{d^{n-1}}{dx^{n-1}}(x^2-1)^n\right)x^{k-1}dx$$

$$\because\ \frac{d^{n-1}}{dx^{n-1}}(x^2-1)^n=0$$

No3の結果をもちいて

$$Q\equiv\sum_{k=0}^{n-1}\left({}_{n-1}C_k\right)\frac{n!}{(n-(n-1-k))!}\frac{n!}{(n-k)!}(x+1)^{n-(n-1-k)}(x-1)^{n-k}$$

$$=\sum_{k=0}^{n-1}\left({}_{n-1}C_k\right)\frac{n!}{(k+1)!}\frac{n!}{(n-k)!}(x+1)^{k+1}(x-1)^{n-k}$$

$$Q(1)=\square(1+1)^1(1-1)^n+\cdots+\square(1+1)^{(n-1)+1}(1-1)^{n-(n-1)}=0$$

$$Q(-1)=\square(-1+1)^1(-1-1)^n+\cdots+\square(-1+1)^{(n-1)+1}(-1-1)^{n-(n-1)}=0$$

4章演習問題[9]
No5　(1) の部分積分　その2

$$\int_{-1}^{1}\left(\frac{d^n}{dx^n}(x^2-1)^n\right)x^k dx = -k\int_{-1}^{1}\left(\frac{d^{n-1}}{dx^{n-1}}(x^2-1)^n\right)x^{k-1}dx$$

その1を書く

同じ手続きをもう1回くりかえす

$$=(-k)(-k+1)\int_{-1}^{1}\left(\frac{d^{n-2}}{dx^{n-2}}(x^2-1)^n\right)x^{k-2}dx$$

積分の中のx^kが$x^0=1$になるまで

$$=(-1)^k k!\int_{-1}^{1}\left(\frac{d^{n-k}}{dx^{n-k}}(x^2-1)^n\right)dx = 0 \qquad (k<n)$$

4章演習問題[9]
No6　(2)

- P_nもP_mも多項式
- m＞nのときは，⑵の左辺は，ルジャンドル関数P_mとx^nの積，P_mとx^{n-1}の積などの積分になる．
- (1)の結果から，すべての項がゼロ．
- m＝nのときだけ計算すれば十分．
- あとは教科書の解答へ

4章の演習問題[10 (b)]

$$f(x) = \frac{1}{1+x^2}$$

$$x \quad = 0, \tfrac{1}{8}, \tfrac{1}{4}, \tfrac{3}{8}, \tfrac{1}{2}$$

$$x^2 \quad = 0, \tfrac{1}{64}, \tfrac{1}{16}, \tfrac{9}{64}, \tfrac{1}{4}$$

$$f(x) = 1, \tfrac{64}{65}, \tfrac{16}{17}, \tfrac{64}{73}, \tfrac{4}{5}$$

$$\int_0^{1/2} f(x)dx \sim \frac{\frac{1}{8}}{3}\left\{1 + 4\times\tfrac{64}{65} + 2\times\tfrac{16}{17} + 4\times\tfrac{64}{73} + 1\times\tfrac{4}{5}\right\}$$

$$= \frac{1}{24}\{1 + 3.938 + 1.882 + 3.507 + 0.8\} = \frac{11.127}{24}$$

$$\sim 0.4636$$