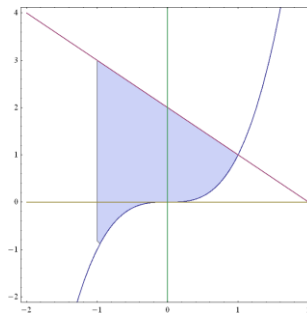


1. $D = \{(x, y) | 1 \leq x \leq 2, 3 \leq y \leq 4\}$ のとき, $\iint_D e^{3x+5y} dx dy$ の値を求めよ.

$$\begin{aligned} \iint_D e^{3x+5y} dx dy &= \iint_D e^{3x} e^{5y} dx dy = \int_1^2 e^{3x} dx \times \int_3^4 e^{5y} dy = \frac{1}{3}(e^6 - e^3) \times \frac{1}{5}(e^{20} - e^{15}) \\ &= \frac{1}{15} e^{18} (e^3 - 1)(e^5 - 1) \end{aligned}$$

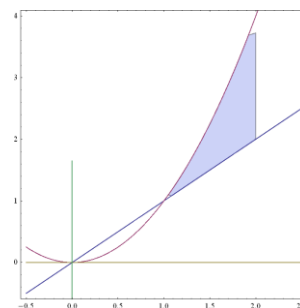
2. 曲線 $y = x^3$ と直線 $y = 2 - x$ および $x = -1$ で囲まれた領域 D における積分 $\iint_D xy dx dy$ を求めよ.
領域のグラフを描くこと (以下同様).

$$\begin{aligned} \iint_D xy dx dy &= \int_{-1}^1 dx \left(\int_{x^3}^{2-x} dy xy \right) = \int_{-1}^1 dx \left(x \left(\int_{x^3}^{2-x} dy y \right) \right) \\ &= \int_{-1}^1 dx x \frac{1}{2} \{(2-x)^2 - x^6\} = \int_{-1}^1 \frac{1}{2} \{4x - 4x^2 + x^5 - x^7\} dx \\ &= 2 \times \frac{1}{2} \int_0^1 (-4x^2) dx = -\frac{4}{3} \end{aligned}$$



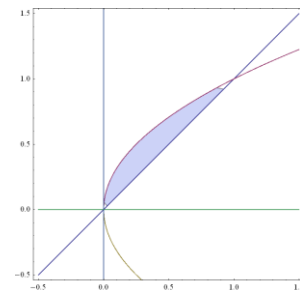
3. 領域 $D = \{(x, y) | 1 \leq x \leq 2, x \leq y \leq x^2\}$ のとき $\iint_D xy dx dy$ の値を求めよ.

$$\begin{aligned} \iint_D xy dx dy &= \int_1^2 dx \int_x^{x^2} dy xy = \int_1^2 dx x \int_x^{x^2} dy y = \int_1^2 dx x \left\{ \frac{1}{2} (x^4 - x^2) \right\} \\ &= \left[\frac{1}{2} \left\{ \frac{1}{6} x^6 - \frac{1}{4} x^4 \right\} \right]_1^2 = \frac{16}{3} - \frac{1}{12} - 2 + \frac{1}{8} = 5 + \frac{3}{12} - 2 + \frac{1}{8} \\ &= 3 + \frac{9}{24} = 3 + \frac{3}{8} = \frac{27}{8} \end{aligned}$$



領域 $D = \{(x, y) | y^2 \leq x \leq y, 0 \leq y \leq 1\}$ のとき $\iint_D xy dx dy$ の値を求めよ.

$$\begin{aligned} \iint_D xy dx dy &= \int_0^1 dy \int_{y^2}^y dx xy = \int_0^1 dy y \int_{y^2}^y dx x = \int_0^1 dy \frac{y}{2} \{y^2 - y^4\} \\ &= \frac{1}{8} - \frac{1}{12} = \frac{1}{24} \end{aligned}$$



4. 一辺が 1m の正方形を底面とし, 高さが 1m, 2m, 2m, 3m の高さをもつ図形の体積を求めよ.

$$\begin{aligned} V &= \iint_D f(x, y) dx dy = \iint_D (x + y + 1) dx dy = \int_0^1 dx \int_0^1 dy (x + y + 1) = \\ &= \int_0^1 dx \left(xy + \frac{y^2}{2} + y \right)_0^1 = \int_0^1 dx \left(x \cdot 1 + \frac{1}{2} + 1 \right)_0^1 = \int_0^1 \left(x + \frac{3}{2} \right) dx = \left[\frac{1}{2} x^2 + \right. \\ &\left. \frac{3}{2} x \right]_0^1 = \frac{1}{2} + \frac{3}{2} = 2 \end{aligned}$$

