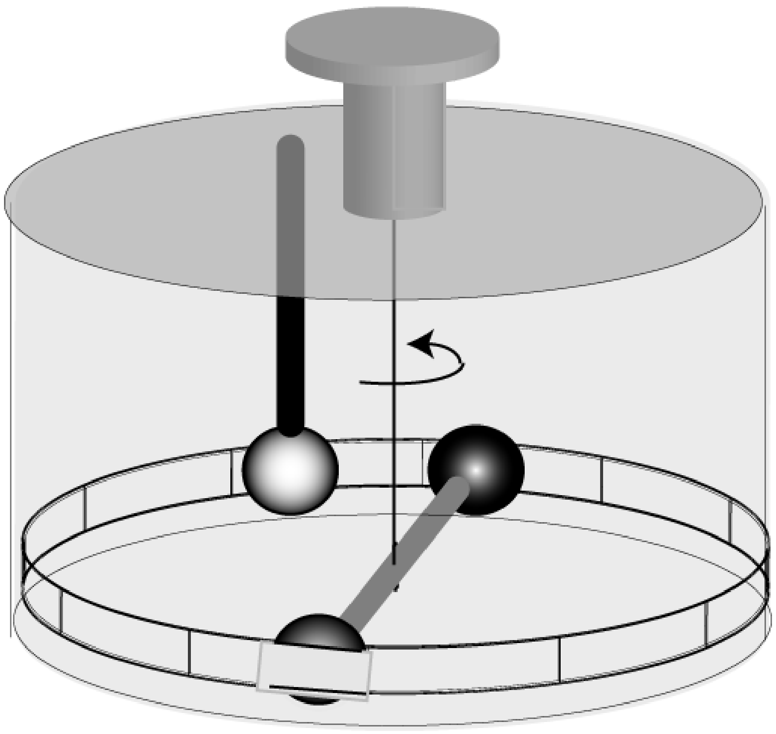
電気的な力
クーロンの実験

$$F = k\frac{qQ}{r^2}$$



- 点電荷 *q, Q* に比例
- 反発力($qQ > 0$)と引力($qQ < 0$)
- 距離*r* の2乗に反比例(逆2乗則)
- 媒質により異なる比例定数*k*

$$k_0 \approx 8.98755\times10^9\ \mathrm{N\cdot m^2/C^2} \approx 9.0\times10^9\ \mathrm{N\cdot m^2/C^2}$$

真空の誘電率 ε_0

- 真空中のクーロン力

$$F = k_0 \frac{qQ}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$$

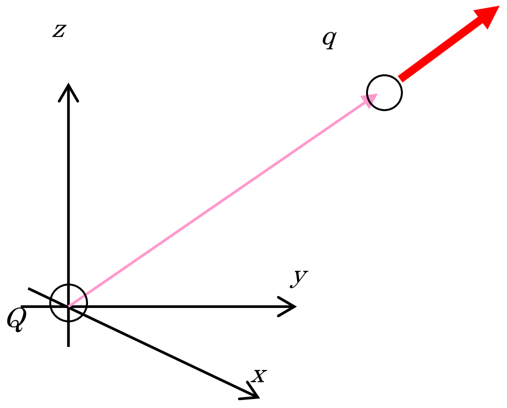
$$k_0 = \frac{1}{4\pi\varepsilon_0} \simeq 9.0 \times 10^9 \text{ N m}^2/\text{C}^2$$

真空の誘電率

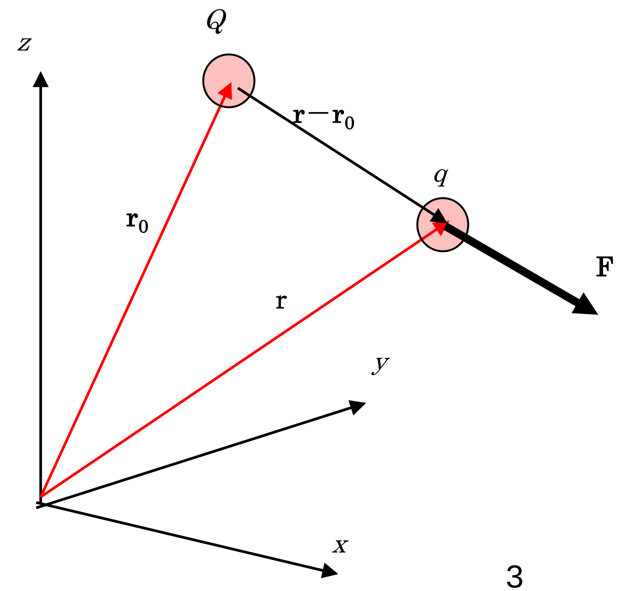
$$\varepsilon_0 = \frac{1}{4\pi k_0} \simeq 8.9 \times 10^{-12}\,\text{C}^2/\text{Nm}^2$$

２個の点電荷の間のクーロン力

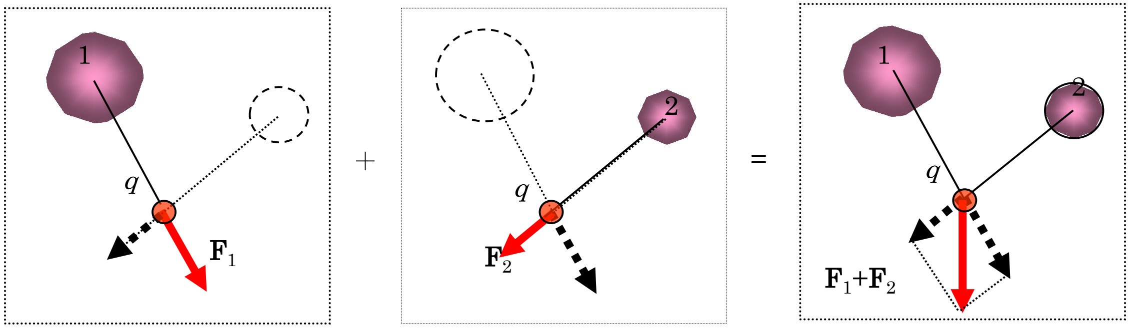
$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\left(\frac{\vec{r}}{r}\right),\ r = |\vec{r}|$$



$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{|\vec{r}-\vec{r}_0|^2}\left(\frac{\vec{r}-\vec{r}_0}{|\vec{r}-\vec{r}_0|}\right)$$



多数の点電荷によるクーロン力
重ね合わせの原理



$$\vec{F}=\vec{F}_1+\vec{F}_2=k_0\frac{qQ_1}{|\vec{r}-\vec{r}_1|^2}\left(\frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|}\right)+k_0\frac{qQ_2}{|\vec{r}-\vec{r}_2|^2}\left(\frac{\vec{r}-\vec{r}_2}{|\vec{r}-\vec{r}_2|}\right)$$

$$=\frac{1}{4\pi\varepsilon_0}q\sum_{j=1,2}\frac{Q_j}{|\vec{r}-\vec{r}_j|^2}\left(\frac{\vec{r}-\vec{r}_j}{|\vec{r}-\vec{r}_j|}\right)$$

$$\vec{F}=\vec{F}_1+\vec{F}_2+\cdots+\vec{F}_N$$

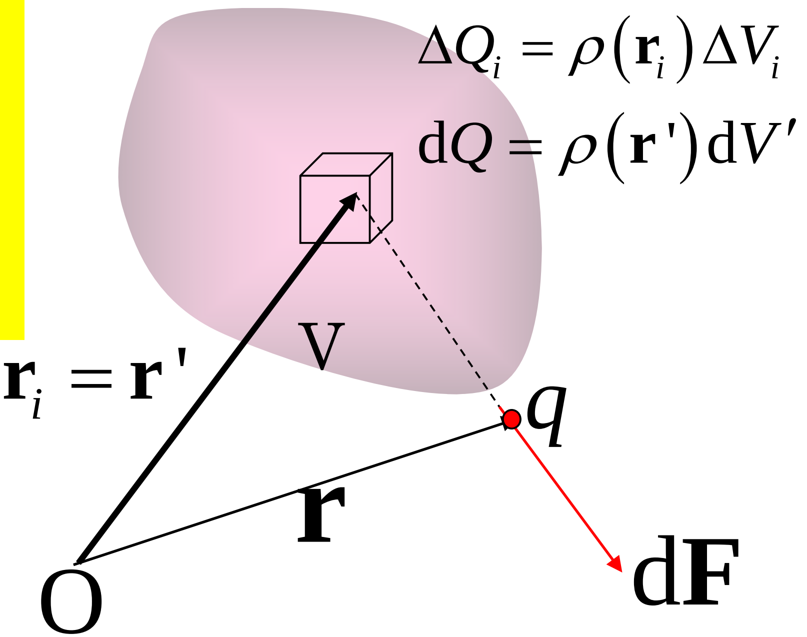
$$=\frac{1}{4\pi\varepsilon_0}q\sum_{j=1\cdots N}\frac{Q_j}{|\vec{r}-\vec{r}_j|^2}\left(\frac{\vec{r}-\vec{r}_j}{|\vec{r}-\vec{r}_j|}\right)$$

連続な電荷分布によるクーロン力(*)

$$\Delta\vec{F}_j = k_0\, q \frac{\Delta Q_j}{|\vec{r}-\vec{r}_j|^2} \frac{\vec{r}-\vec{r}_j}{|\vec{r}-\vec{r}_j|}$$

$$= k_0 q \frac{\rho(\vec{r}_j)\Delta \mathrm{V_j}}{|\vec{r}-\vec{r}_j|^2} \frac{\vec{r}-\vec{r}_j}{|\vec{r}-\vec{r}_j|}$$

$$d\vec{F} = k_0 q \frac{\rho(\vec{r}')dV'}{|\vec{r}-\vec{r}'|^2} \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}$$



$$\vec{F}(\vec{r}) = \iiint_V d\vec{F} = \frac{1}{4\pi\varepsilon_0} q \iiint_V \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|^2} \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|} dV'$$

クーロン力の成分表示（＊）

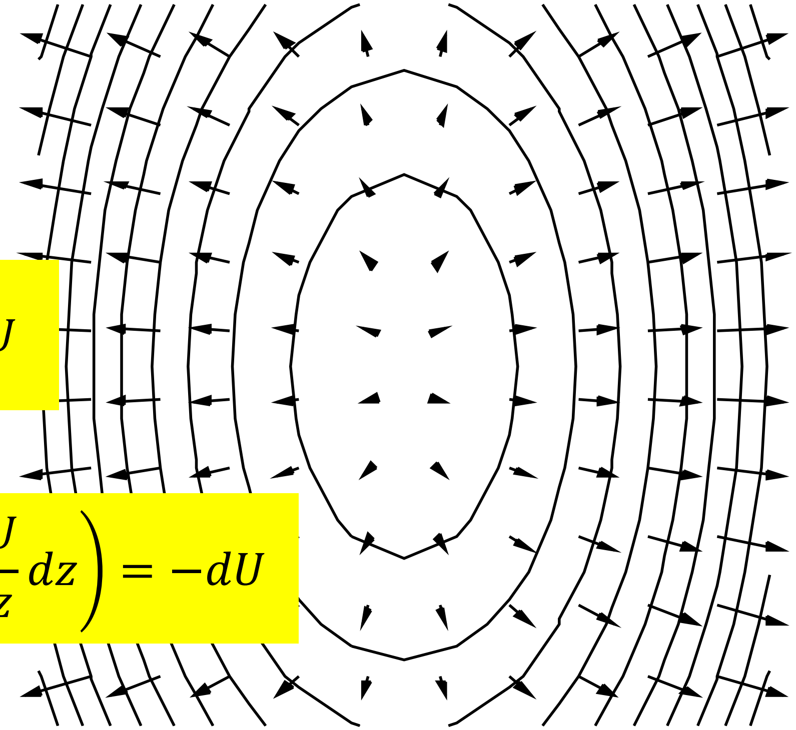
$$\vec{F}(\vec{r}) = \iiint_V d\vec{F} = \frac{q}{4\pi\varepsilon_0}\iiint_V \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|^2}\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}dV'$$
$$= \frac{q}{4\pi\varepsilon_0}\iiint_V \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|^3}(\vec{r}-\vec{r}')dV'$$

$$|\vec{r}-\vec{r}'| = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$$

$$F_x(x,y,z) = \frac{q}{4\pi\varepsilon_0}\iiint_V \frac{\rho(x',y',z')}{\left\{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}\right\}^3}(x-x')dx'dy'dz'$$
$$F_y(x,y,z) = \frac{q}{4\pi\varepsilon_0}\iiint_V \frac{\rho(x',y',z')}{\left\{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}\right\}^3}(y-y')dx'dy'dz'$$
$$F_z(x,y,z) = \frac{q}{4\pi\varepsilon_0}\iiint_V \frac{\rho(x',y',z')}{\left\{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}\right\}^3}(z-z')dx'dy'dz'$$

保存力、仕事、位置エネルギー

$$F = -\frac{dU}{dx}$$



$$\vec{F} = \left(-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right) = -\mathrm{grad}\ U = -\nabla\ U$$

$$\vec{F}\cdot d\vec{r} = -\nabla U \cdot d\vec{r} = \left(-\frac{\partial U}{\partial x}dx - \frac{\partial U}{\partial y}dy\ - \frac{\partial U}{\partial z}dz\right) = -dU$$

山（位置エネルギー）の斜面の傾斜（グラディエント）
の符号を変える（山を降る向きにする）と力になる
力の向き＝山の等高線と直交する向き

クーロン力の位置エネルギー

原点にある点電荷Q、無限遠を基準とした位置エネルギー

$$U(\vec{r}) = k\frac{qQ}{r} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r}$$

$$\frac{\partial U}{\partial x} = -kqQ\frac{x}{r^3},\quad \frac{\partial U}{\partial y} = -kqQ\frac{y}{r^3},\quad \frac{\partial U}{\partial z} = -kqQ\frac{z}{r^3}$$

$$\vec{F} = -\nabla U = \left(-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right)$$

$$= kqQ\left(\frac{x}{r^3}, \frac{y}{r^3}, \frac{z}{r^3}\right) = k\frac{qQ}{r^3}\vec{r} = k\frac{qQ}{r^2}\left(\frac{\vec{r}}{r}\right)$$

原点以外にある点電荷Q

$$U(\vec{r}) = k\frac{qQ}{|\vec{r}-\vec{r}_0|} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{|\vec{r}-\vec{r}_0|}$$

位置エネルギーの重ね合わせ

$\vec{F}_j = -\nabla U_j \quad (j = 1,2,\cdots,N)$とすると

$$U = U_1 + U_2 + \cdots + U_N \iff \begin{aligned}\vec{F} &= -\nabla U = -\nabla(U_1 + U_2 + \cdots + U_N) \\ &= -\nabla U_1 - \nabla U_2 \cdots - \nabla U_1 \\ &= \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N\end{aligned}$$

$$U(\vec{r}) = \frac{1}{4\pi\varepsilon_0} q \sum_{j=1,N} \frac{Q_j}{|\vec{r} - \vec{r}_j|}$$

$$U(\vec{r}) = \frac{1}{4\pi\varepsilon_0} q \iiint_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV'$$